

Hong Kong Mathematics Olympiad (2010 / 2011)

Heat Event (Individual)

香港数学竞赛 (2010 / 2011)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 求 2^{2011} 除以 13 的余数。

Find the remainder when 2^{2011} is divided by 13 .

2. 已知 $x^2 + y^2 = 1$ ，求 $2x + 5y^2$ 的极大值。

Given that $x^2 + y^2 = 1$, find the maximum value of $2x + 5y^2$.

3. 已知 $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$ 及 $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$ ，求 ab 的值。(答案以根式表示)

Given that $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$ and $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$, find the value of ab . (Give your answer in surd form)

4. 在 $\triangle ABC$ 内，分别垂直于三条边 AB 、 BC 及 CA 的高的比是 $3 : 4 : 5$ 。若三条边的长均为整数，求 AB 的最小值。

In $\triangle ABC$, the ratio of the altitudes perpendicular to the three sides AB , BC and CA is $3 : 4 : 5$. If the lengths of the three sides are integers, find the minimum value of AB .

5. 整数 x 减去 12 后是一个整数的平方。将 x 加上 19 后则是另一个整数的平方。求 x 的值。

An integer x minus 12 is the square of an integer. x plus 19 is the square of another integer. Find the value of x .

6. 甲、乙及丙三人互相传球。甲首先将球传出。有多少不同方案使得经过 5 次传球后，球会回传给甲？

A , B and C pass a ball among themselves. A is the first one to pass the ball to the other one. In how many ways will the ball be passed back to A after 5 passes?

7. 求 $\sqrt{7-\sqrt{12}-\sqrt{13-2\sqrt{12}}}$ 的值。

Find the value of $\sqrt{7-\sqrt{12}-\sqrt{13-2\sqrt{12}}}$.

8. 学校推出每张面值为 \$10、\$15、\$25 及 \$40 的四种卖物券。甲班用若干张 \$100 纸币买了 30 张卖物券，包括其中两种卖物券各 5 张及另外两种卖物券各 10 张。问甲班共享了多少张 \$100 纸币购买卖物券？

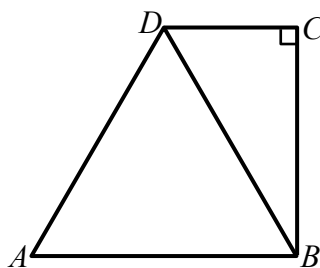
A school issues 4 types of raffle tickets with face values \$10, \$15, \$25 and \$40. Class A uses several one-hundred dollar notes to buy 30 raffle tickets, including 5 tickets each for two of the types and 10 tickets each for the other two types. How many one-hundred dollar notes does Class A use to buy the raffle tickets?

9. 某长方形的长和阔均为整数。若面积比周界大 9，求周界的值。

The length and the width of a rectangle are integers. If its area is larger than its perimeter by 9, find the perimeter.

10. 如图二， $ABCD$ 为一个梯形，其中 $\angle C = 90^\circ$ 。若等边三角形 ABD 的面积为 $16\sqrt{3}$ ，求梯形 $ABCD$ 的面积。

In Figure 2, $ABCD$ is a trapezium with $\angle C = 90^\circ$. If the area of the equilateral triangle ABD is $16\sqrt{3}$, find the area of trapezium $ABCD$.



图二

Figure 2